# Development and Validation of a Computational Model for Home Built Solar Water Heating Systems

A Thesis

Presented in Partial Fulfillment of the Requirements

for Undergraduate Distinction

By

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## ABSTRACT

Uncertainty over future sources of energy provides strong motivation to explore technologies that free systems from traditional energy inputs. This project investigated home built solar water heating systems that can be used in northern climates. A computational model was developed to assist homeowners in the design of their own system. A prototype was designed using the model and then constructed. The prototype was used to validate the predictions given by the model. An economic analysis of installing a solar hot water heating system was then carried out.

## ACKNOWLEDGMENTS

My Family - Who allowed a study of a home built system to actually be home built.

Dr. Seppo Korpela - Who helped me realize that there is more gratifying work in the world then building race cars.

## TABLE OF CONTENTS

## LIST OF TABLES

## LIST OF FIGURES

# CHAPTER 1

# INTRODUCTION

Many technologies have been developed to free houses from traditional energy dependency [1]. When implemented, these technologies often affect the layout and structure of a house. Some technologies are also very sophisticated and costly. As fuel prices rise, the cost of the construction of a new building by traditional methods is inflating rapidly. These facts make the development of sustainable technologies that can be easily retrofitted onto existing buildings a worthy pursuit.

Space and water heating account for approximately 65 % of the total energy use in an average home in the United States [3] [11]. A solar hot water heater uses the energy from the sun to directly heat water. These systems can be retrofitted with relative ease onto existing structures. The main component of a solar hot water system is the collector, which allows incident solar energy to heat a fluid. An example of a flat plate solar thermal collector can be found in Figure 1.1 [10]. Incident solar energy enters the insulated collector through a glazed covering and increases the temperature of the absorber plate. The absorber plate is made of a material with a high thermal conductivity, usually copper. An array of tubes is attached to the plate and fluid is pumped through them. The circulating liquid in the tubes is heated by the hot absorber plate. Both the absorber plate and the tube array are usually painted black.

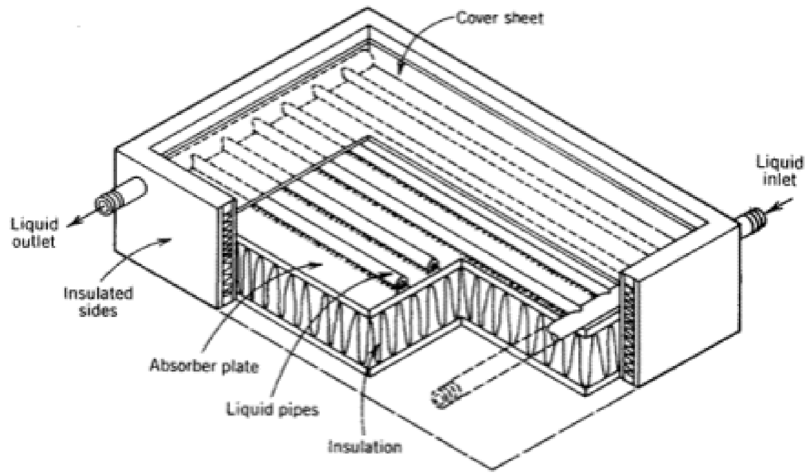


Figure 1.1: Flat Plate Solar Thermal Collector

Two main types of solar hot water systems are available, open loop and closed loop. An open loop system circulates potable water in a solar thermal collector. A closed loop system circulates an anti-freeze solution in the collector and uses a heat exchanger to warm potable water. Open loop systems are less costly then closed loop systems, but they cannot be used in climates where the temperature drops below freezing for a significant amount of time. Closed loop systems can be used in freezing climates, and are the topic of this thesis. It is worthwhile to study systems that are usable in colder climates since many home built solar hot water solutions are already available for climates that are suitable for open loop systems.

A closed loop solar water heating system can be seen in Figure 1.2 [4]. The working fluid of the system is warmed in the collector and travels into a heat exchanger. After leaving the heat exchanger, the working fluid is pumped back into the collector. The potable water is kept separate from the working fluid in a storage tank. The tank allows the heated water to accumulate, but also contains the heat exchanger and an auxiliary heater. The heat exchanger facilitates energy transfer between the collector working fluid and potable water. An auxiliary heater is necessary for the system to maintain a usable hot water temperature in unfavorable conditions. This thesis investigates systems with natural gas auxiliary heat. Many commercial examples of solar water heating systems are available. The cost of a closed loop system begins at approximately \$3000. Professional installation of these systems also adds significant cost to the project. However, a motivated homeowner can construct and install an effective system without commercial components.

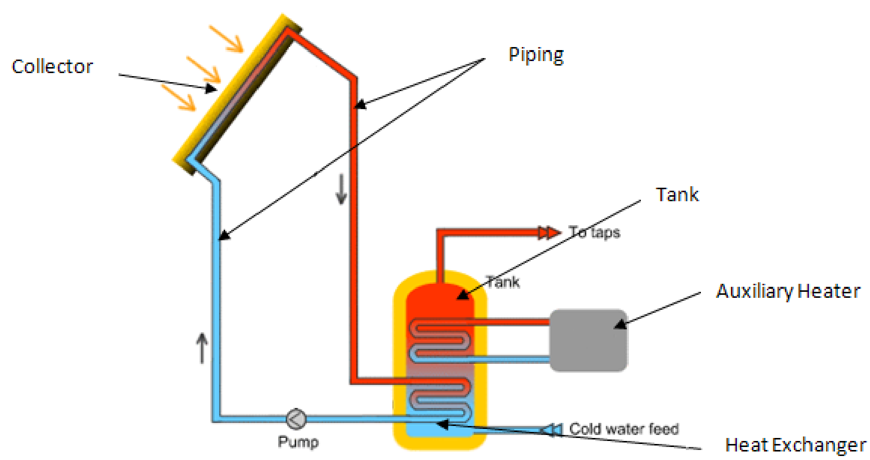


Figure 1.2: Closed Loop Solar Water Heating System

A rigorous analysis of solar swimming pool heaters was published by de Winter [5] in 1975. The publication included details of collector sizing, design, and construction. De Winter developed an expression for the steady state output of a solar thermal collector. This expression can be seen in Equation 1.1 below. In the expression, $F_3$

is the collector efficiency, Ac is the area of the collector, $\alpha$ is the absorptivity of the radiation coating, $Q_i$ is the amount of incident solar radiation, $U_l$ is the loss coefficient, $T_{C_i}$ is the temperature of the incoming fluid, and $T_a$ is the ambient temperature. The efficiency term was developed in three steps, beginning with the fin efficiency of the absorber plate. The Florida Solar Energy Center published a detailed guide on the general applications of solar thermal systems [2] in 2006. This manual provides guidelines for the implementation of solar thermal collectors for potable hot water use.

$$Q_{coil} = F_3 Ac\left(\alpha Q_i - U_l(T_{c_i} - T_a)\right) \tag{1.1}$$

The goal of this thesis was to develop a computational model to simulate the performance of a closed loop solar hot water heating system. After formulating equations, a Matlab and Simulink program was created to model a year of use of a solar hot water system. The model also computed the economic consequences of installing a proposed system. A prototype system was constructed to validate the results produced by the model. Future work on the project includes integrating an optimization routine into the system model.

# CHAPTER 2

# THEORY

## 2.1 Formulation of Equations

A closed loop solar hot water system is comprised of three thermally significant components. These include a collector, piping, and a tank with an integral heat exchanger. A governing equation was developed for each component. Two notable assumptions were made while developing these relationships. First, that the specific heat of fluids is constant. Second, that the collector and tank fluid are at a uniform temperature. The thermally lumped analysis of the collector and tank are considered appropriate because of the high thermal conductivity of the collector material and the mixing action in the tank. Of the components, the collector and tank have a time and temperature dependency. The time dependency comes from the fact that these two components have a large thermal capacity. The piping is assumed to have no thermal capacity, and therefore only has a temperature dependency. Table 2.1 shows the unknowns and functional dependency for the collector and tank. The two components are first order systems, but they are coupled, so the entire system is second order.

Table 2.1: Unknowns and Functional Dependency of Components

| Component | Unknowns | Functional Dependency |
|---|---|---|
| Collector | Lumped Collector Temperature<br>Outlet Fluid Temperature | Time<br>Inlet Fluid Temperature |
| Tank | Lumped Tank Temperature<br>Outlet Fluid Temperature | Time<br>Inlet Fluid Temperature |

The governing expression for the solar thermal collector was developed using the results of Equation 1.1 [5]. With the assumption of constant specific heat, the steady state temperature increase of circulating fluid can be seen in Equation 2.1 below. In the expression, $\dot{m}_h$, $C_{p_{hf}}$, and $T_{c_o}$ are the mass flow rate, specific heat,and temperature of fluid leaving the collector. Equation 2.2 results when the time dependent energy storage of the collector material is incorporated with Equation 2.1. In Equation 2.2,$m_c$, $C_{p_c}$, and $T_c$ are the mass, specific heat, and temperature of the collector itself. Equation 2.2 was employed as the governing expression for the solar thermal collector.

$$\dot{m}_h C_{p_{hf}} (T_{c_o} - T_{c_i}) = F_3 Ac\,(\alpha Q_i - U_l (T_{c_i} - T_a)) \tag{2.1}$$

$$m_c C_{p_c} \frac{dT_c}{dt} = \dot{m}_h C_{p_{hf}} T_{c_o} + F_3 \alpha Q_i Ac - (F_3 U_l Ac - \dot{m}_h C_{p_{hf}}) T_{c_i} + F_3 U_l Ac T_a \tag{2.2}$$

The development of the $U_l$ term in Equation 2.2 relies on heat transfer coefficient correlations as developed in Korpela [7]. The correlations used for the convective heat transfer coefficients within the collector airspace, for the forced convection on the surfaces of the collector, and between the working fluid and collector tubing can be seen in Equations 2.3, 2.4, and 2.5. In these equations, Ra, Pr, and Re are the dimensionless Rayleigh, Prandtl, and Reynolds numbers. Additionally, $H_c$ and $W_c$

are the height and width of the enclosed cavity of the collector.

$$Nu = 0.42 Ra^{0.25} Pr^{0.012} \left(\frac{H_c}{W_c}\right)^{-3} \tag{2.3}$$

$$Nu = (0.37\, Re^{4/5} - 871) Pr^{1/3} \tag{2.4}$$

$$Nu = 3.66 + \frac{0.0688(D/L)RePr}{1 + .04[(D/L)RePr]^{2/3}} \tag{2.5}$$

Like the collector, the expression for the tank fluid temperature was developed using a thermally lumped technique. The expression for tank temperature in Equation 2.6 was developed first. In this expression, the $m_t$, $C_{p_w}$, and $T_t$ are the mass, specific heat, and temperature of the fluid within the hot water tank. Furthermore, $R_t$ is the thermal resistance of the tank, $\dot{m}_t$ is the mass flow of water leaving the tank, and $T_{fw}$ is the temperature of feed water ending the tank. The $Q_{hex}$ term accounts for the energy input from the heat exchanger.

$$m_t C_{p_w} \frac{dT_t}{dt} + \left(\frac{1}{R_t} + \dot{m}_t C_{p_w}\right) T_t = Q_{hex} + \frac{T_a}{R_t} + \dot{m}_t C_{p_w} T_{fw} \tag{2.6}$$

The temperature drop for the fluid on the collector side of the heat exchanger was found using the expression for heat transfer in pipe flow with a constant wall temperature. The assumption of constant wall temperature was made because of the high thermal conductivity of the tubing material. The expression for the temperature drop can be seen in Equation 2.7. In this expression, $T_{h_i}$, $T_s$, and $T_{c_o}$ are the temperatures of fluid entering the heat exchanger, the pipe wall, and fluid exiting the collector. Additionally, $P, L$, and $U$ are the wetted perimeter, length, and heat transfer coefficient of the pipes. The rate of heat transfer from the heat exchanger with the assumption constant wall temperature is found in Equation 2.8. Equation 2.9 results upon combining and rearranging equations 2.6 and 2.8. Equation 2.9 was employed as the governing expression for the tank.

$$T_{h_i} = T_s - \exp\left(\frac{-PLU}{\dot{m}_h C_{p_{hf}}}\right)(T_s - T_{c_o}) \tag{2.7}$$

$$Q_{hex} = \dot{m}_h C_{phf}\left[(T_t - T_{hi}) - (T_t - T_{ho})\right] \tag{2.8}$$

$$m_t C_{pw}\frac{dT_t}{dt} + \left(\frac{1}{R_t} + \dot{m}_t C_{pw}\right)T_t = \frac{T_a}{R_t} + \dot{m}_t C_{pw} T_{fw} + \dot{m}_h C_{phf}\left(T_{ho} - T_{hi}\right) \tag{2.9}$$

Energy loss was considered from the piping in the collector loop between the collector and heat exchanger. The temperature of the working fluid after piping loss is given as in Equation 2.7 above. Again, the uniform wall temperature assumption was made because of the high thermal conductivity of the pipe walls. The losses encountered between the tank and the hot water use point were not taken into account. When hot water is used in a home, the temperature is often regulated by the mixing of hot and cold water streams. This was taken into account with Equation 2.10 below. In the expression, $\dot{m}_t$ and $\dot{m}_u$ are the mass flow rates of fluid leaving the tank and being used. Temperature of the hot water is denoted by $T_u$. The mass flow of the hot water leaving the tank is dependent on the tank temperature because the $T_u$ term is constant. This relationship has the effect of decreasing the mass flow of water from the tank when its temperature is hot in comparison to the use temperature.

$$\dot{m}_t = \frac{\dot{m}_u\left(T_{fw} - T_u\right)}{\left(T_{fw} - T_t\right)} \tag{2.10}$$

## 2.2 Economics

The economic payoff of installing a solar hot water system was investigated using the principles of engineering economics as presented in Arora [6]. An investment is considered viable if it possesses a positive net present value, or NPV. A positive NPV results if the return on investment for the initial capital is greater than the minimum acceptable rate of return, or MARR. The MARR is framed as the interest rate that is expected for an investment of a certain risk level. To find present value, the expected future cost savings on natural gas must be discounted at the MARR rate. This is carried out using Equation 2.11 below. The present value of each year's expected savings is then summed, and the initial investment of the system is subtracted. Because the price of natural gas does not stay constant, the expected yearly cost savings from the system fluctuates. The future cost of natural gas can be found using the current price and estimations for the annual percentage rate increase. After rearranging, Equation 2.11 is also used for this task. In the expression, $PV, FV, i$, and $n$ are the present value, future value, interest rate, and compounding periods of an investment.

$$PV = FV\left(1+i\right)^{-n} \tag{2.11}$$

The sizing of a solar hot water system provides an economic optimization problem. The output of the collector is much greater in the summertime than the wintertime. This is because the amount of solar radiation is higher in the summer, as is the ambient temperature. Figure 2.1 below illustrates this fact. In the figure, the normalized system output is plotted for a year. The time scale is that of months, with 1 corresponding to January and 12 corresponding to December. As can be seen from the plot, if the solar system was designed to provide 100% of the hot water for a home in the middle of the winter, the system would have large a amount of unutilized capacity in the summer. The unutilized capacity is a waste of the capital investment of the system. Thus, an economically optimum system will require some degree of auxiliary heat.

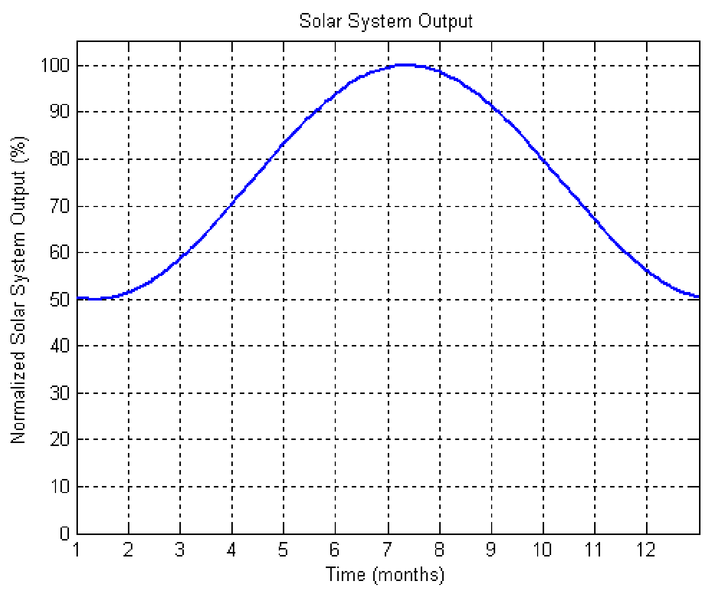


Figure 2.1: Normalized Solar System Output for a Year

# CHAPTER 3

# SYSTEM MODELING

A model of the system was constructed using Matlab and Simulink. The model relies on the equations presented above after they were transformed using the Laplace method. The model executes in the following fashion and a graphic of this can be seen in Figure 3.1 below.

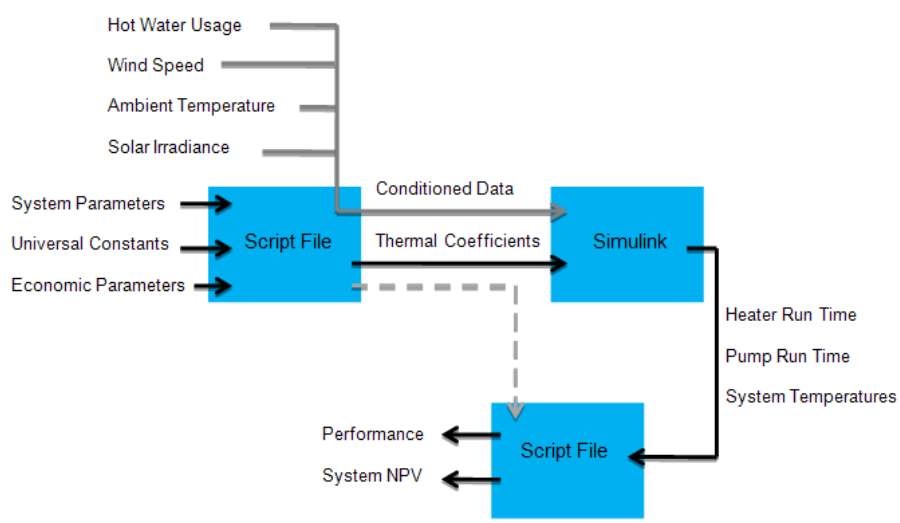


Figure 3.1: Schematic of Solar Water Heating System Model

The specifications for the system are input into a script file. Every aspect of the system can be altered. This includes for example, the heat exchanger pipe size, the thermal conductivity of the tank insulation, and the spacing between tubes within the collector. Similarly fluid property information and universal constants are included. Economic parameters such as the raw material costs for the system and the MARR are also included in the file. Environmental parameters such as yearly profiles of solar radiation and ambient temperature, and the daily hot water use cycle for system are specified as well.

Execution of the script file results in the calculation system parameters from the input data. These dependent parameters mainly include heat transfer coefficients, but other values such as the head loss for the piping system are calculated. With the necessary parameters calculated, the script file calls a Simulink program. The Simulink block diagram can be found in Appendix A. The Simulink program simulates the operation of the collector for an entire year. It iterates twice, as the first year is simulated to find the initial conditions for the actual execution of the model. The outputs for the simulation are yearly temperature profiles for different parts of the system. Most importantly though, the simulation gives as an output the amount of time that the auxiliary heater was required to run.

With the amount of time that the auxiliary heater was required to run, the script file calculates the amount of natural gas that the system consumed. The model then compares the amount of natural gas used to what would have been used if the solar hot water system were not present. With the yearly savings, the net present value of the investment is determined for the simulated system at different annual changes in natural gas prices. The script file also produces plots of fluid temperatures in the system throughout the year and the values of the dependent system parameters.

The output of the model for a generic system for the first day of July can be seen in Figure 3.2 below. The aggressive pulse in the inlet and outlet temperatures of the collector corresponds to when the sun is out. The collector temperatures are low during the night time. The sawtooth-like temperature oscillations in the system originate from the use of hot water from the tank. When hot water is drawn from the tank, the temperature in the tank decreases when cold feed-water is fed into the tank. As the tank temperature drops, the amount of heat claimed from the collector loop increases. This causes the collector loop temperatures to decrease. The tank temperature does not fluctuate as violently as the collector temperatures for two reasons. First, the thermal capacity of the tank is larger than the collector. Second, the tank is better insulated and sheltered than the collector.

Generic results for the model over the entire year are shown in Figure 3.3 below. The figure gives the output of the model with no auxiliary heat. It is obvious that energy is far more scarce in the wintertime than in the summertime. The quantized nature of the plot results from the fact that average monthly climate data was used as the environmental input. With input from an auxiliary heater, the areas of the tank temperature curve that are below the use temperature will simply be pushed up. The area of the temperature curve that is below the use temperature is related to the amount of auxiliary heat needed.

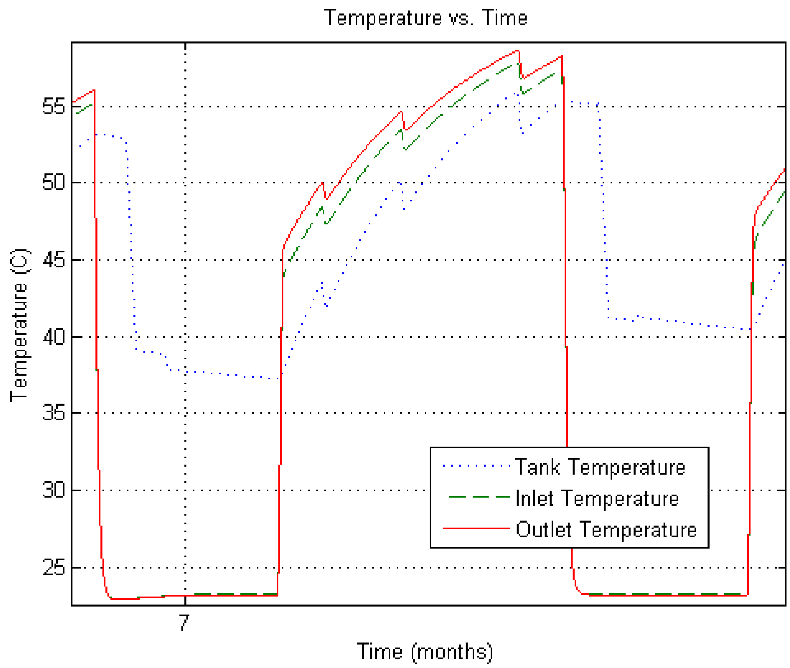


Figure 3.2: Temperature Profiles for a one Day Simulation

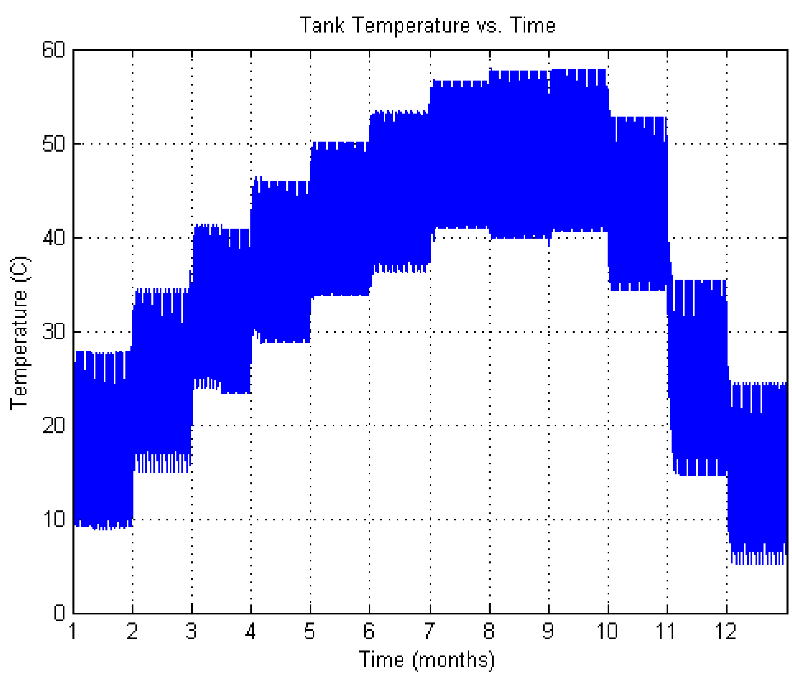


Figure 3.3: Tank Temperature for a one Year Simulation, Without Auxiliary Heat

# CHAPTER 4

# PROTOTYPE CONSTRUCTION

A prototype system, for use in Columbus OH, was designed using the model. The system was designed to provide hot water for a household of two people who adjusted their hot water use cycle to better accommodate a solar system. This mainly includes showering in the evening when the temperature in the tank is highest. The daily hot water use cycle for the prototype system can be seen in Table 4.1 below. The system was sized to conform to constraints placed on it by the building on which the system was to be installed. This resulted in a suboptimal system, but this is of no consequence for the validation of the model. The total cost of the system can be seen in Table 4.2 below. The collector area is equal to $4.31m^2$. The copper plate and piping account for a large portion of the system cost. Although significant, the cost of the prototype system is approximately 50% lower then a comercial system of similar collector area.

The prototype system was constructed in a residential driveway over the course of three weekends. Figure 4.1 shows the completed solar thermal collector. In the figure, the black absorber plate and collector fluid tubes are visible. The collector is insulated and glazed. Figure 4.2 shows the heat exchanger for the system. The heat exchanger used was a homemade tube-in-tube type using a flexible hot water tank

Table 4.1: Hot Water Use Cycle for Prototype System

| Time | Gallons | Use |
|---|---|---|
| 7:00 AM | 2 | Wash Hands and Face, Shave |
| 1:00 PM | 2.5 | Wash Hands and Food Preparation |
| 5:00 PM | 3.2 | Washing Hands and Laundry/ Food Prep |
| 9:00 PM | 15 | Showering |
| 11:00 PM | 0.5 | Washing Hands |

Table 4.2: Prototype System Cost

| Component | Cost ($) |
|---|---|
| Copper Plate | 698.36 |
| Natural Gas Hot Water Tank | 346.32 |
| Copper Tube | 277.81 |
| Polycarbonate Sheet | 228.92 |
| Circulation Pump | 219.98 |
| Pipe Couplings | 199.74 |
| Valves and Instrumentation | 164.27 |
| Insulation | 160.56 |
| Wood Framing Materials | 65.00 |
| Total System | 2360.90 |

supply line for the inner tube. The ridges on the tube increased turbulence in the system, which improves the heat transfer.

Figure 4.3 shows the interface between the heat exchanger, tank, and collector. The pipe coming from the upper right of the figure brings heated fluid from the collector to the heat exchanger. The circulation pump is along this section of pipe. The pipe partially shown in the upper left is the return to the collector. The tank is plumbed to the heat exchanger to allow the natural circulation of the water in the tank through the heat exchanger. This deviates from the situation assumed during

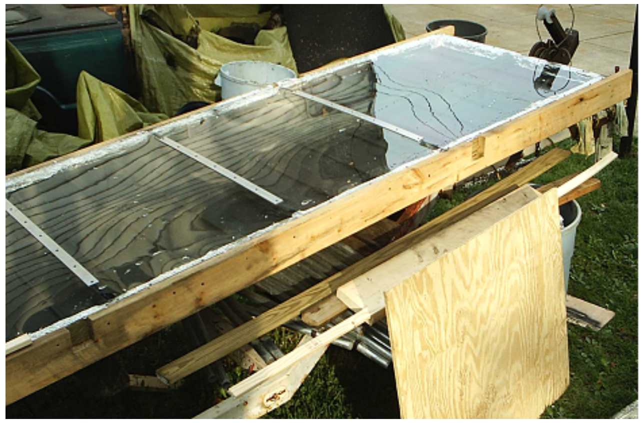

Figure 4.1: Solar Thermal Collector

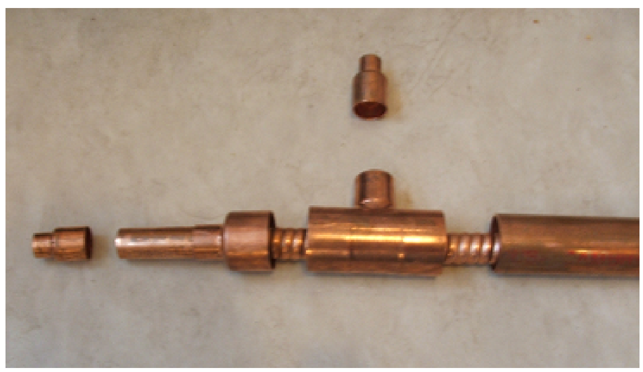

Figure 4.2: Heat Exchanger

the formulation of the governing equations, but the results are not unduly affected. Figure 4.4 shows the completed system. The system was installed on a recycled boat trailer to allow for easy transportation.

Figure 4.3: Heat Exchanger and Tank Interface

Figure 4.4: Complete Prototype System

# CHAPTER 5

# MODEL VALIDATION

The prototype was used to validate the system model. During testing, the amount of solar radiation, the ambient temperature, and the average wind speed were collected using a Davis VP2 weather station. Temperatures were collected using fluid thermometers that were plumbed into the system. The collector was initially tested independent from the rest of the system. This data was collected on two separate days. Figure 5.1 shows the simulated and experimental responses for the solar collector. The dynamic behavior of the prototype was captured by the model. The response of the collector was relatively quick, having reached steady state in approximately 4 minutes. The steady state error of the device was calculated to be 5.3%. A second trial was made, and similar results were acquired. For the second trial, the steady state error was found to be 4.6%. This is a satisfactory level of agreement between the model and prototype. This test also allowed the efficiency of the collector to be calculated. The collector was found to be approximately 62%, which is comparable to commerical flat plate collectors.

The entire system was then tested and simulated. Figure 5.2 shows the simulated and experimental responses for the entire system. This test took place over the course of 4 hours. The experimental and simulated behavior correlates well, even though

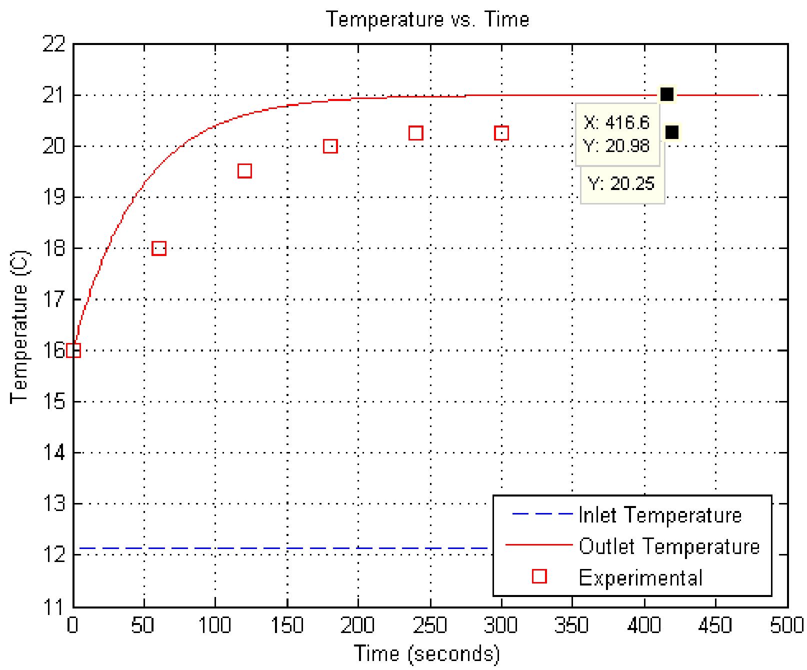


Figure 5.1: Dynamic Response of Collector Only

the experimental system appeared to have reached steady state while the simulated system had not. A contributor to the non steady state behavior of the model is the fact that all fluid properties were assumed to be constant during the formulation of equations. The sudden drop in the temperature in the simulated system corresponds to a 3 gallon draw from the hot water tank. The experimental system did not directly show the effects of the hot water use, mainly because the temperature readings were taken every 15 minutes. The system regains the temperature lost from the draw within the time span. These results are considered satisfactory though, especially for a thermally lumped analysis.

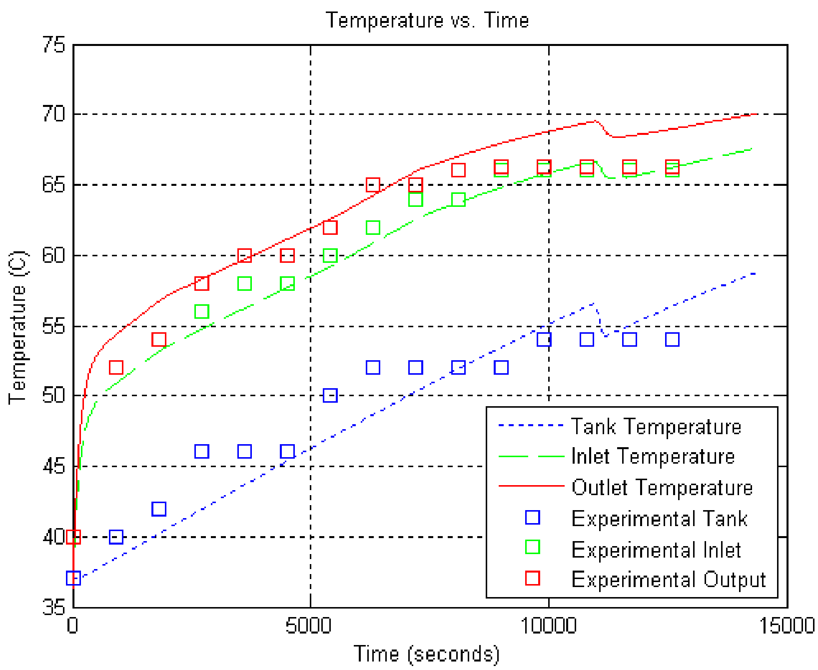


Figure 5.2: Dynamic Response of Complete System

# CHAPTER 6

# ECONOMIC ANALYSIS

The net present value of the prototype system was evaluated using the model. A MARR of 4.0% was chosen. This corresponds to a low risk financial investment after adjustment for inflation. The yearly benefit of the system is strongly dependent on the price of natural gas. If the price of natural gas is high, the economic benefit of a solar hot water system is large. Figure 6.1 below shows the NPV of the prototype system against the annual percentage cost increase of natural gas. If natural gas prices do not rise from their current rates, The net present value of the prototype system would be a loss of \$321.31. Instead, if natural gas prices were to increase at a rate of 10% per year, the net present value of the system would be \$2698.21.

The sizing of a solar hot water system presents an economic optimization problem. Although many system specifications affect the performance of the device, the main variable is the size of the collector. This determines the amount of auxiliary heat input that is required to maintain a usable tank temperature. The amount of auxiliary heat input determines the annual cost savings from installing a system. A large collector will allow a system to capture more energy and reduce the amount of auxiliary heat that is needed, but a large collector increases the initial investment. A smaller collector reduces initial costs, but increases the amount of auxiliary heat

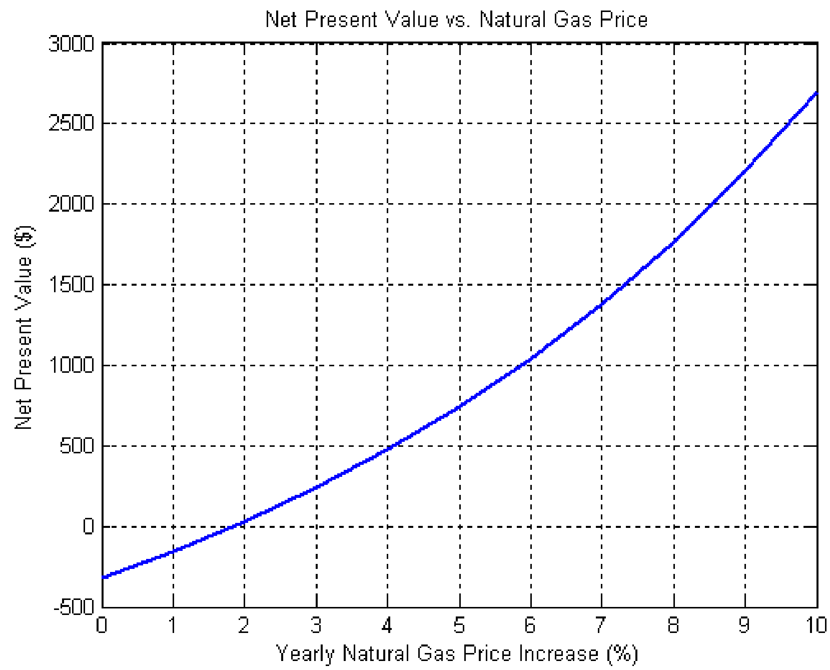


Figure 6.1: Prototype System Net Present Value and Natural Gas Yearly Price

that is needed. The optimum amount of auxiliary heat depends on the rate of annual natural gas price increases. Figure 6.2 below illustrates this fact. The plot shows NPV against collector area for different levels of annual natural gas price increases. A higher annual increase of natural gas prices would make a larger system optimum. A smaller annual increase would make a smaller system optimum.

It is interesting to note that the net present value of all systems considered are negative if current natural gas prices decrease or stay constant. The payback is not negative for constant natural gas prices if an analysis that ignores the time value of money is carried out. This type of analysis is often used by proponents of solar hot water heating, but arrives at erroneous results. It has been shown though that constructing a homemade solar hot water heating system can be a sound economic investment in the face of the rising costs of fossil fuels [8]. Additionally, the price of

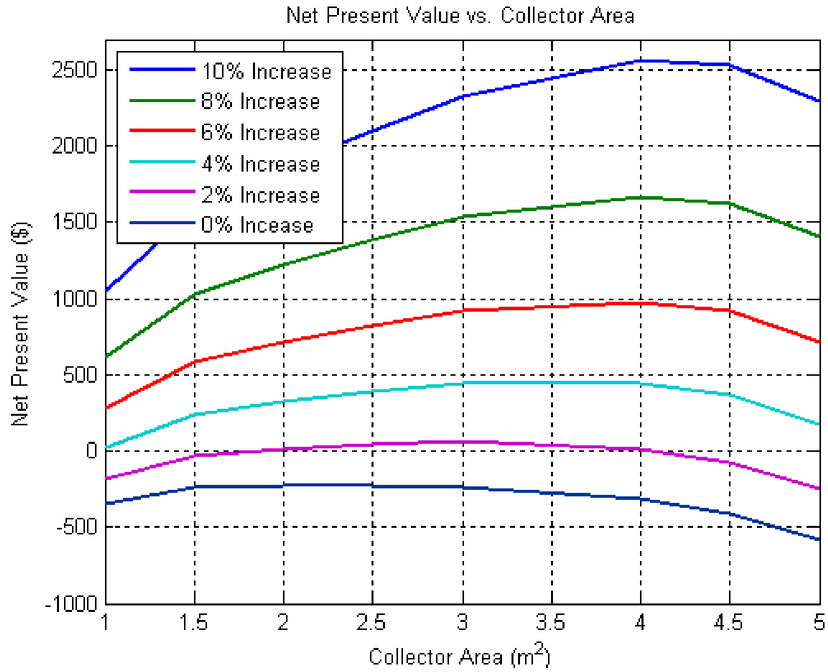


Figure 6.2: Net Present Value and Collector Area

copper is increasing at a rapid rate, so it is a prudent activity for one to make an investment in such a system now, while the prices are reasonable [9].

Figure 6.2 above shows a relatively simple relationship between the area of the collector and the NPV. When all of the system parameters are considered, this analysis becomes extremely complicated. Future work on the project will consist of integrating an automated optimization script into the model to help homeowners design a system on their own. This program will be incorporated into an easy to use graphical user interface and distributed freely. This should provide motivated homeowners the tools necessary to design an optimized system for their geographic location and hot water use cycle.

# APPENDIX A

# SIMULINK MODEL

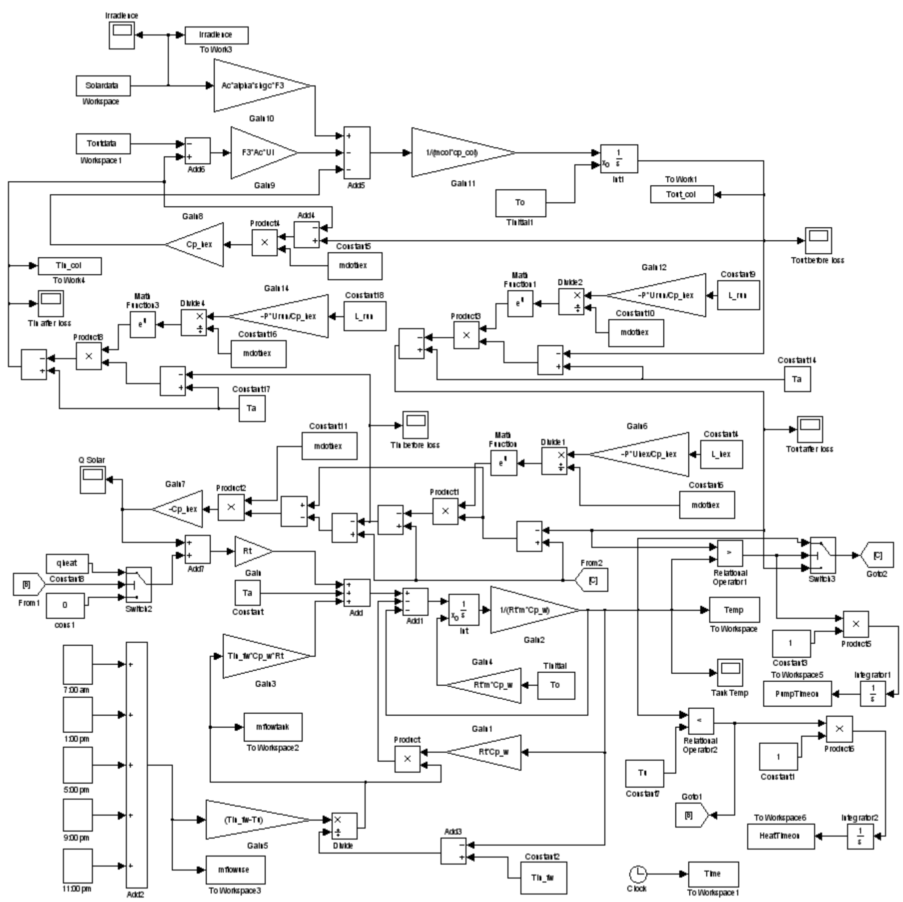

Figure A.1: Simulink Block Diagram

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